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Nonlinear model for the self-interference compensation in AEM

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Summary

This study presents an analysis of existing methods for compensating for carrier interference in AEM data, culminating in the proposal of a new approach based on a polynomial expansion of the self-interference field based on the measured field of two dipoles. Carrier interference compensation algorithms using third-order polynomials were developed, demonstrating that the use of a third-order polynomial provides a compensation depth comparable to the sensitivity of the field sensors used. Experimental verification confirmed effective self-interference compensation using data obtained with a two-dipole configuration. The geometric limitations inherent in this method were identified, and an optimal regularization parameter was introduced and determined for estimating the coefficients of a cubic self-interference model, further enhancing the reliability and accuracy of the compensation process.

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Introduction

The primary source of interference from the carrier frame is caused by uncontrolled eddy currents induced in conductive elements. When field sources are mounted on an aircraft such as fixed or rotary wing aircraft, these currents can generate a field reaching approximately 1% of the primary field (Vovenko et al. 2017). This presents a significant challenge when further separating the secondary and primary fields, considering the field from the carrier frame as part of the primary field. Interference from the carrier frame dynamically depends on the relative geometry between the transmitter and receiver. Compensation for this interference is often achieved by adjusting the installation geometry, which requires solving a positioning problem to determine the relative coordinates of the receiver and transmitter. Specifically, this problem involves estimating the dipole parameters based on the characteristics of the generated field (Smith, 2001; Tkhorenko et al. 2015).
Modern AEM survey systems typically describe the transmitter field as a dipole field (Zhdanov, 2017). Carrier interference is considered as a linear form in terms of specific measured parameters (Vovenko et al. 2017). However, for new systems characterized by increasing receiver sensitivity and decreasing transmitter-receiver distances (Moilanen, 2022), there is a need to improve standard models. This paper describes a new approach to solving the problem of self-interference compensation. It is tested on data from EM4H (Vovenko et al. 2013), a frequency-domain AEM system widely used in modern surveys in Russia.

Theory and Method

To begin, here is a brief description of the idea presented by Karshakov et al. (2023). Let the interfering signal source have a dipole magnetic moment vector ΔM. This vector can be considered constant due to the rigidity of the transmitter frame. The interfering field ΔH is not constant due to the flexibility of the cable and varies depending on the relative position of the transmitter and receiver. The idea of the compensation method was originally based on the geometric ideas described by Tkhorenko et al. (2015). The key point is that all field sources have the same coordinates. In this case, the magnetic field for any dipole source can be calculated as

$$\mathbf{H} = \frac{1}{4\pi|\mathbf{R}|^3}\left(3\mathbf{e}_\mathbf{R}\mathbf{e}_\mathbf{R}^\mathrm{T} - I\right)\mathbf{M} = \Omega(\mathbf{R})\mathbf{M}, \tag{1}$$

where $\mathbf{e_R} = \mathbf{R}/|\mathbf{R}|$ is the unit vector defining the direction of the radius vector $\mathbf{R}$ between the transmitter and the receiver, $\mathbf{I}$ is the 3x3 identity matrix. Considering any dipole $\mathbf{M}_i$ installed in the same point, we can be sure that the field $\mathbf{H}_i$ is calculated in the same way (1). If the interference field appears at the same point, its field $\Delta\mathbf{H}$ is related to the unknown $\Delta\mathbf{M}$ by the same formula (1).
Obviously, given three linearly independent vectors $\mathbf{M}_0$, $\mathbf{M}_1$, $\mathbf{M}_2$ we can find $\Delta\mathbf{M}$ as a linear combination, and from (1) it follows that $\Delta\mathbf{H}$ is the same combination of $\mathbf{H}_0$, $\mathbf{H}_1$, $\mathbf{H}_2$. Karshakov et al. (2023) proved that

$$\Delta\mathbf{H} = k_0\mathbf{H}_0 + k_1\mathbf{H}_1 . \tag{2}$$

Note, that the interference is a complex vector $\Delta\mathbf{H} \in \mathbb{C}^3$. According to this compensation method, the vectors $\mathbf{H}_0$, $\mathbf{H}_1 \in \mathbb{R}^3$, since they are derived from the in-phase field component. Therefore, the coefficients in expression (2) are complex and are estimated at the compensation stage using the least-squares method (LSM), k_0, $k_1 \in \mathbb{C}^{3x3}$. Matrix coefficients are used because they allow for additional compensation for receiver measurement errors, such as non-orthogonality of the sensitivity axes and unequal scale factors.
Here, we explore the possibility of improving the compensation quality by switching to a nonlinear model. Given the linear model (2), the self-interference field can be more generally represented as a polynomial expansion of the field vectors $\mathbf{H}_0$, $\mathbf{H}_1$:

$$\Delta\mathbf{H} = P_n\left(\mathbf{H}_0, \mathbf{H}_1\right) \tag{3}$$

Where P_n is a polynomial of degree n. The polynomial coefficients are approximated by LSM during calibration at a sufficiently high altitude, where the quadrature component of the response is minimized, and the in-phase component is reduced to a uniform value, for example, measured at the lowest frequency:

$$min\left\{\sum_{k=1}^{K}\| \, Im\mathbf{H}_{0i} - ImP_n(*)\|_2\right\},$$
$$min\left\{\sum_{k=1}^{K}\| \, Re\left(\mathbf{H}_{0i} - \mathbf{H}_{00}\right) - ReP_n(*)\|_2\right\}, \quad (4)$$

where the second index corresponds to each frequency of the primary field. Clearly, as the degree of the polynomial increases, so does the number of parameters in the model (3). While the linear model contains 18 complex coefficients, the second-degree polynomial has 81 coefficients, and the third-degree polynomial contains 249 coefficients. However, the set of measurements from which these coefficients are to be estimated remains the same. With such a large number of unknown parameters, the task of determining them can become ill-conditioned, and the model can compensate not only for the self-interference but also for internal noise or some global external disturbances. Therefore, regularization is necessary.

The idea is to consider an additional constraint on the coefficients of the polynomial in expressions (4). Let the vector $\mathbf{p} = \{p_0, p_1, \ldots,\}$ represent the set of coefficients of a polynomial. A penalty term of the following form is added to the functional being minimized:

$$min\left\{\sum_{k=1}^{K}\| \, Im\mathbf{H}_{0i} - ImP_n(*)\|_2 + \alpha\sum_{j=1}^{m}p_j^{\,2}\right\},$$
$$min\left\{\sum_{k=1}^{K}\| \, Re\left(\mathbf{H}_{0i} - \mathbf{H}_{00}\right) - ReP_n(*)\|_2 + \alpha\sum_{j=1}^{m}p_j^{\,2}\right\}, \quad (5)$$

where α is the regularization parameter. To select the regularization parameter α, we use the approach proposed by Tikhonov et al. (1995). According to this approach, the residual threshold is set at the noise level. If the residual exceeds the noise level, undercompensation occurs; if it falls below the noise level, overcompensation occurs.

It is also important to note that the proposed model (3) has geometric limitations. Assume that the carrier interference dipole is shifted to a point O' relative to the origin, where the primary and additional dipoles are located. Let l denote the distance between the origin and the shifted dipole, and R' be the new radius vector. Then the expression for the radius vector will be as follows:

$$\frac{1}{|\mathbf{R}'|^3} = \sum_{n=2}^{\infty}\frac{n(n-1)}{2}\frac{l^{n-2}}{|\mathbf{R}|^{n+1}} \quad (6)$$

Formula (1) in this case takes the form:

$$\mathbf{H}' = \frac{1}{4\pi}\left(\sum_{n=2}^{\infty}\frac{n(n-1)}{2}\frac{l^{n-2}}{|\,\mathbf{R}\,|^{n+1}}\right)\left(3\mathbf{e}_\mathbf{R}\mathbf{e}_\mathbf{R}^{\mathrm{T}} - I\right)\mathbf{M} \quad (7)$$

The series for the field contains all integer powers of $1/|\mathbf{R}|$ starting with 3. At the same time, the expression for the point dipole field contains only the third power, $1/|\mathbf{R}|^3$. Therefore, any expression containing positive integer powers of the components of the magnetic induction vector of a point magnetic dipole will contain only powers of the form $3n$, $n \in \mathbb{Z}$. In other words, powers 4, 5, 7, etc., which are present in the expression for $\mathbf{H}'$, will be absent from this expansion.

Nevertheless, for a set of measurements at a fixed radius $|\mathbf{R}| \approx$ const and various angles, the cubic approximation yields a smaller error compared to the linear approximation (Fig. 2a – raw signals, 2b – linear model, 2c – cubic model). It should also be noted that at points near 116,000 corresponding to a small angle between the main dipole and the radius vector, the linear algorithm actually yields poor results.

Overall, the algorithm performs well at a roughly constant distance, which largely corresponds to the actual EM4H system configuration, as the tow cable is always taut during flight. However, if, for

example, the transmitter and receiver were located on different aircrafts with significantly varying distances between them, the model would become inappropriate.

Examples

The quadrature components of the vertical field component were analyzed for four different frequencies (130 Hz, 520 Hz, 2080 Hz, and 8320 Hz). Overcompensation occurred during the fitting of the cubic model coefficients, resulting in distortions at all frequencies, which are clearly observed in the 30,000–31,000 m interval (Fig. 2c, right panel). This occurred despite the fact that the cubic model adequately describes high-amplitude interference near sample 3000 in the calibration zone (Fig. 2c, left panel). Therefore, a regularization parameter was introduced during the coefficient fitting stage.

Figure 1 shows the dependence of the residuals calculated at each frequency in the calibration zone on the value of the regularization parameter α. The optimal value of the parameter α was selected based on an estimate of the receiver noise level at the corresponding frequency.

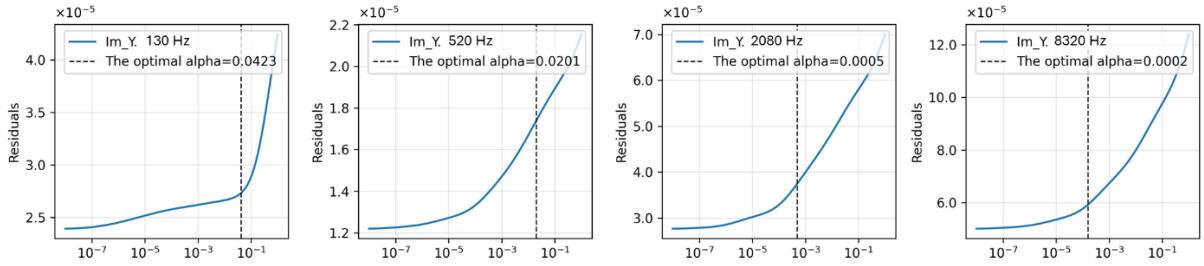


Figure 1 *Optimal regularization parameter.*

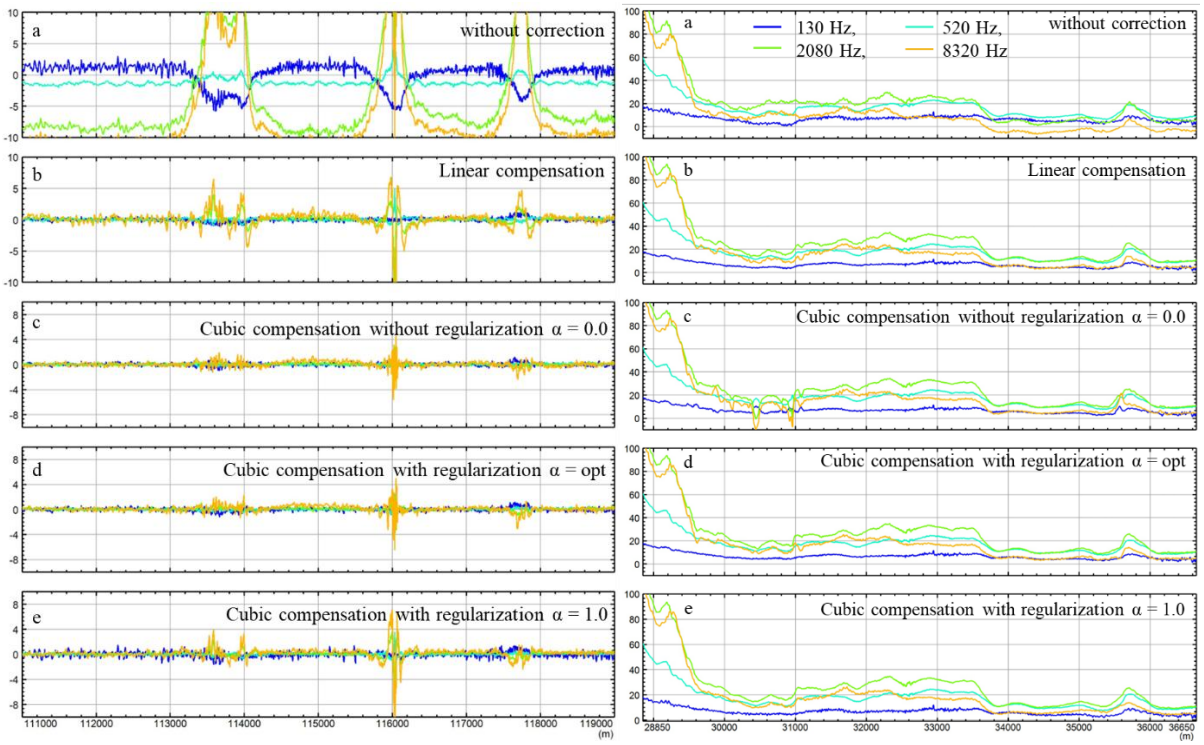


Figure 2 *Compensation stage(left) and survey profile(right). Quadrature component: (a)- without compensation, (b) – linear compensation, (c) – cubic polynomial without regularization, (d) – cubic polynomial with optimal regularization parameter, (e) – cubic polynomial with regularization parameter* α = *1*

Figure 2 shows, from top to bottom, a comparison of the residual noise at the compensation stage (left side) and the responses obtained at the survey height (right side) without correction (Fig. 2a), after

linear compensation (Fig. 2b), compensation with a cubic polynomial without regularization (Fig. 2c), with the optimal parameter α (Fig. 2d), and with the regularization parameter α = 1 (Fig. 2e). It is clear that introducing the regularization parameter leads to compensation that is better than the linear polynomial, but leaves more residual noise than the non-regularized cubic polynomial. Introducing regularization at the survey height eliminates overcompensation.
With the optimal regularization parameter, the residuals correspond to the system noise level. Near 31,000 m in the right panel, high-frequency measurement distortions are clearly visible (orange line); they are significantly smaller for the regularized cubic model, but significantly larger for the non-regularized cubic model. Furthermore, near 29,000 m, additional low-frequency noise (blue line) is clearly visible, arising from excessive regularization (α = 1).

Conclusion

This study presents an analysis of existing methods for compensating for carrier interference in AEM data, culminating in the proposal of a new approach based on a polynomial expansion of the self-interference field based on the measured field of two dipoles. Carrier interference compensation algorithms using third-order polynomials were developed, demonstrating that the use of a third-order polynomial provides a compensation depth comparable to the sensitivity of the field sensors used. Experimental verification confirmed effective self-interference compensation using data obtained with a two-dipole configuration. The geometric limitations inherent in this method were identified, and an optimal regularization parameter was introduced and determined for estimating the coefficients of a cubic self-interference model, further enhancing the reliability and accuracy of the compensation process.

Acknowledgements

We sincerely appreciate the contribution of Andrey Volkovitsky, the author of the three-axis linear model, who suggested considering a polynomial approach. We are also grateful to Maxim Tkhorenko, who pointed out that we likely have geometric limitations.

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